Is True of – Part 2 Linguistic Analog

This month’s post is part 2 of 2 of the exploration of the YouTube video entitled [Russell's Paradox - a simple explanation of a profound problem](https://youtu.be/ymGt7I4Yn3k) by Jeffery Kaplan. What we discussed last month was the summary of Russell’s paradox in which we found that the set of all ordinary sets defined as

\*\*\[ {\mathcal Q} = \{ x | x \mathrm{\;is\;a\;set\;that\;does\;not\;contains\;itself} \} \; \]\*\*

creates a paradox. If we assume ${\mathcal Q}$ does not contain itself (i.e., it is ordinary) then the membership comprehension ‘is a set that does not contain itself’ instructs us that ${\mathcal Q}$ in fact does contain itself. Alternatively, if we assume that ${\mathcal Q}$ does contain itself (i.e., it is extraordinary) then membership comprehension instructs us that it doesn’t.

This type of self-referential paradox mirrors other well-known paradoxes that arise in linguistics such as the [liar's paradox](http://aristotle2digital.blogwyrm.com/?p=209) or the concept of the [self-refuting idea](https://en.wikipedia.org/wiki/Self-refuting_idea). What makes Kaplan’s analysis interesting (whether or not it originates with him) is the very strong formal analogy that he draws between the common act of predication that we all engage in nearly continuously and the more abstruse structure of Russell’s paradox, that few among us know or care about.

The heart of Kaplan’s analogy is the explicit mapping of the ‘contains’ idea from set theory – that sets contain members or elements, some of which are sets, including themselves – with the ‘is true of’ idea of predication.

\*image\*

To understand what Kaplan means by the ‘is true of’, we will again follow the structure of his examples with some minor verbal modifications to better suit my own taste.

Predication is the act of saying something about the subject of a sentence by giving a condition or attribute belonging to the subject. In the following sentence

\*\*Frodo is a brave hobbit.\*\*

The subject of the sentence is “Frodo” and the predicate is “is a brave hobbit”. The predicate “is a brave hobbit” is true of Frodo, as anyone whose read Lord of the Rings can attest. Kaplan then points out that the first basic rule of naïve set theory, which he states as

\*\*Rule #1 of sets: there is a set for any imaginable collection of a thing or of things”

has, as its formal analogy in predication, the following:

\*\*“Rule #1 of predication: there is a predicate for any imaginable characteristic of a thing.”\*\*

The two key rules of set theory that lead to Russell’s paradox have their analogs in predication as well.

Rule #10 of sets, which allows us to have sets of sets, is mirrored by Rule #10 of predication that tells us we can predicate things about predicates. As an example of this, consider the following sentence is

\*\*”Is a Nazgul” is a terrifying thing to hear said of someone.\*\*

The predicate “Is a Nazgul” is the subject of that sentence and “is a terrifying thing to hear said of someone” is the predicate.

Rule #11 of sets, which allows sets to contain themselves (i.e., self-reference), finds its analog in Rule #11 of predication that tells us that predicates can be true of themselves.

Here we must proceed a bit more carefully. Let’s start with a simple counterexample:

\*\*”Is a hobbit” is a hobbit.\*\*

This sentence is clearly false as the subject, the predicate “Is a hobbit”, is clearly not a hobbit itself, it is a predicate. But now consider the following sentence, which Kaplan offers:

\*\*”Is a predicate” is a predicate\*\*

This sentence is clearly true as the subject, the predicate “Is a predicate”, is clearly a predicate. And, so, Rule #11 of predication works.

Kaplan then constructs a table similar to the following (again only minor verbal tweaks are done for the predicates that are not true of themselves to suit my own taste)

|  |  |
| --- | --- |
| Predicates not true of themselves | Predicates true of themselves |
| “is a brave hobbit” | “is a predicate” |
| “is a Nazgul” | “is a string of words” |
| “keeps his oaths” | “typically comes at the end of a sentence” |

Note that the predicate “is true of itself” is a predicate that is true of all the predicates that are true of themselves, that is to say, of all the predicates that can be placed in the right column of the table above. The next step is then to ask what is the predicate of all the predicates that can be placed in the left column of the above table. A little reflection should satisfy oneself that the predicate “is not true of itself” fits the bill.

The final step is to ask in which of the two column does “is not true of itself” fall, or, in other words,

\*\*is “is not true of itself” true of itself?\*\*

If we assume that it is true of itself then the content found between the quotes tells us that it is not true of itself. Equally vexing, if we assume that it is not true of itself that assumption matches the content found between the quotes tell us that it is true of itself. In summary: if it is then it isn’t and if it isn’t then it is. And we’ve generated the predicate analogy to Russell’s paradox.

Of course, this is just a form of the well-known Liar’s Paradox so we might be willing to just shrug it off as a quirk of language but I think Kaplan is making a deeper point that is worth deeply considering. At the root of his analysis is the realization that there are objective rules (or truths), that these rules generate self-referential paradoxes, and, so, one is forced to recognize that paradoxes are an essential ingredient in not just all of language but of thought itself. And no amount of patching, such as was done to naïve set theory, can rescue us from this situation. This observation, in turn, has the profound philosophical implication that there is only so far that logic can take us.